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Electromagnetic catalysis of the radiative transitions of $\nu_i \to \nu_j \gamma$ type in the field of an intense monochromatic wave

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Abstract

The radiative decay of the massive neutrino $\nu_i \to \nu_j \gamma$ in a circularly polarized electromagnetic wave is investigated within the Standard theory with lepton mixing. The decay probability in the wave field does not contain a threshold factor $\sim (1 - m_i/m_j)$ as opposed to the decay probability in a vacuum or in a constant uniform external field. The phenomenon of the gigantic enhancement ($\sim 10^{33}$) of the neutrino decay probability in external wave field is discovered. The probability of the photon splitting into the neutrino pair is obtained.

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1 Introduction

For quite a long time there has been a stable interest in the physics of the massive neutrino. Investigations in this area touch, to a certain extent, the feasible phenomenon of lepton mixing and its manifestations in neutrino physics and, consequently, in astrophysics and cosmology (neutrino oscillations [1] and their connection with the problem of the solar neutrino [2,3], the massive neutrino decay [4] and its influence on the relict radiation spectrum [5], the nature of the Supernova 1987A neutrino outburst [6], β-decay and the problem of the 17-keV neutrino [7].

On the other hand, it is well known that an intensive electromagnetic field can significantly influence the properties of the massive neutrino itself [8] and even induce novel lepton transitions with flavor violation, forbidden in a vacuum [9]. A curious, in our opinion, effect of enhancing influence of the magnetic field on the probability of the radiative decay $\nu_i \to \nu_j \gamma$ ($i \neq j$, i and j enumerate the definite mass neutrino species) of the massive neutrino was discovered in our recent work [10] in the framework of the Glashow-Weinberg-Salam (GWS) theory with lepton mixing. We stress that lepton mixing, similarly to quark mixing, appears quite natural if the neutrinos have a non-degenerate mass spectrum, and, in itself, does not go beyond the framework of the standard electroweak theory.

In addition the development of intensive electromagnetic field generation techniques and the current possibility to obtain waves of high strength of electromagnetic field, namely $\mathcal{E} \sim 10^9 V/cm$, stimulate the investigation of quantum processes in strong external fields. Indeed, the parameter of the wave intensity

$$x_e^2 = -\frac{e^2 a^2}{m_e^2} \tag{1}$$

(where a is the amplitude of wave, m_e is the electron mass and e is an elementary charge) characterizing the effect of the electromagnetic wave should not be neglected.

In the present work we investigate the effect of a circularly polarized wave on radiative decay $\nu_i \to \nu_j \gamma$ in the framework of the GWS theory with lepton mixing.

2 The amplitude of the process

In the lowest order of the perturbation theory, a matrix element of the radiative decay of the massive neutrino in the Feynman gauge is described by diagrams of three types, represented in Fig.1, where double lines imply the influence of the external field. For the propagators of intermediate particles (the W-boson, charged scalar and charged lepton) exact solutions are used of the corresponding wave equations in the field of a monochromatic circularly polarized wave with the four-potential

$$A_\mu = a_{1\mu} \cos\varphi + \xi \, a_{2\mu} \sin\varphi, \qquad \varphi = kx \tag{2}$$

where $k^\mu = (\omega, \vec{k})$ is the four-wavevector; $k^2 = (a_1 k) = (a_2 k) = (a_1 a_2) = 0$, $a_1^2 = a_2^2 = a^2$; the parameter $\xi = \pm 1$ indicates the direction of the circular polarization (left- or rightward). Note that vectors $\vec{a}_1$, $\vec{a}_2$ and $\vec{k}$ form a right-handed coordinate system. Provided that $e\mathcal{E}/m_W^2 \ll 1$, the main contributions are made by the diagrams with the virtual W-boson in Fig.1a and the virtual Z-boson in Fig.1c. We stress, that diagram represented in Fig.1c gives the contribution to the amplitude with $i = j$ only. This is due to the fact that flavour-changing neutral currents are absent in Standard Model.

The S-matrix element of the given process can be represented in the following form:

$$S = S_0 + \Delta S \tag{3}$$

where S_0 is the well known matrix element of the radiative decay of the massive neutrino in vacuum [4], and ΔS is the contribution, induced by the wave field:

$$\Delta S = \frac{i(2\pi)^4}{\sqrt{2E_1 V \cdot 2E_2 V \cdot 2q_0 V}} \sum_{n=-2}^{+2} \mathcal{M}^{(n)} \delta^{(4)}(nk + p_1 - p_2 - q) \tag{4}$$

Here p_1, p_2, q and E_1, E_2, q_0 are the four-momenta and energies of the initial, final neutrinos and photon, respectively. $n = 0, \pm 1, \pm 2$ is the difference between the numbers of absorbed and emitted photons of the wave field.

Note that the matrix element of some process in the field of an electromagnetic wave has usually the form of summation of n type (4), where

$-\infty < n < +\infty$ [11]. That only five values of n in our case are possible is extraordinary and is due to the following reasons. The process $\nu_i \to \nu_j\gamma$ is local with the typical scale $\Delta x \leq 1/m_f$ (m_f is the mass of the virtual fermion). In this case the angular momentum conservation degenerates to spin conservation. Since the total spin of the particles participating in this process is no greater than 2 (1/2 + 1/2 + 1), $\mid n \mid_{\max} = 2$ is the maximum difference between the numbers of absorbed and emitted photons of the external field (the photons of a monochromatic circularly polarized wave have a definite spin $\xi = \pm 1$). The direct calculation supports this conclusion. A similar phenomenon has been discovered before [9] in studies of the effect of a circularly polarized wave on flavor-changing transitions of the massive neutrinos $\nu_i \to \nu_j$ ($i \neq j$) with $\mid n \mid_{\max} = 1$.

Notice that in the uniform constant fields the decay $\nu_i \to \nu_j\gamma$ with $m_i > m_j$ is valid only [10]. This is due to the fact that the energy-momentum conservation law in these fields coincides with one in vacuum. On the other hand, as it follows from expression (4), the external electromagnetic wave field can induce also radiative decay with $m_i \leq m_j$ forbidden without the field. Indeed, from the energy-momentum conservation law in the wave field

$$nk + p_1 = p_2 + q$$

the relation follows

$$m_i^2 - m_j^2 \geq -\, 2n(kp_1)$$

In such a manner, the radiation decay $\nu_i \to \nu_j\gamma$ with $m_i \leq m_j$ is possible on condition that $n > 0$.

The exact invariant amplitudes $\mathcal{M}^{(n)}$ in the expression (4) have cumbersome forms and will be published elsewhere. Here we present the amplitude of this process in a physically more interesting case of the ultrarelativistic neutrinos ($E_\nu \gg m_\nu$). In this case the amplitudes $\mathcal{M}^{(n)}$ corresponding to $n \leq 0$ are suppressed by the factor of the small neutrino mass ν_i and the other amplitudes are significantly simplified and may be represented as follows:

$$\mathcal{M}^{(+1)} \;\simeq\; \frac{G_F e^2 a^2}{2\sqrt{2}\pi^2}\,(jk)\,\frac{(\tilde{f}^* F)}{(kq)^2}\Big\{\sum_\ell \Big(K_{i\ell}K_{j\ell}^* - \frac{1}{2}\,\delta_{ij}\Big)J_1(m_\ell)$$

$$
\begin{aligned}
& + & \frac{3}{2}\,\delta_{ij}\sum_q \Big(2T_{3q}Q_q^4\Big)J_1(m_q)\Big\},\\
\mathcal{M}^{(+2)} & \simeq & -\,\frac{G_F}{16\sqrt{2}\pi^2}\,(jFq)\,\frac{(f^*F)}{(kq)^2}\Big\{\sum_\ell\Big(K_{i\ell}K^*_{j\ell}+\frac{1}{2}\,\delta_{ij}\,g_\ell\Big)J_2(m_\ell)\\
& - & \frac{3}{2}\,\delta_{ij}\sum_q\Big(Q_q^3 g_q\Big)J_2(m_q)\Big\}, \qquad (5)\\
j_\mu & = & \bar\nu_j(p_2)\gamma_\mu(1+\gamma_5)\nu_i(p_1),\\
F_{\mu\nu} & = & e(k_\mu a_\nu - k_\nu a_\mu), \qquad a_\mu=(a_1+i\xi\,a_2)_\mu,\\
f_{\mu\nu} & = & e(q_\mu\varepsilon_\nu - q_\nu\varepsilon_\mu), \qquad \tilde f_{\mu\nu}=\frac{1}{2}\,\varepsilon_{\mu\nu\alpha\beta}f_{\alpha\beta},\\
g_f & = & 2T_{3f}-4Q_f\sin^2\theta_W, \qquad f=\ell,q,
\end{aligned}
$$

where index ℓ indicates charged leptons ($\ell = e,\mu,\tau$) and index q indicates quark flavours ($q = u,c,t,d,s,b$), T_{3f} is the third component of the weak isospin and Q_f is the electric charge in units of the elementary charge, ε_μ is the polarization four-vector of the photon, m_ℓ and m_q are the masses of the virtual lepton and quark, respectively, $K_{i\ell}$ is the unitary lepton mixing matrix which can be parameterized similarly to the quarks mixing Kabayashi-Maskawa matrix,

$$
\begin{aligned}
J_1(m_f) & = & \int_0^1\frac{dy}{1-y^2}\int_0^\infty d\tau\,\tau^2 j_0(j_0^2+j_1^2)\,\exp(-i(\Phi(m_f)+\tau)),\\
J_2(m_f) & = & \int_0^1 dy\int_0^\infty d\tau\,\tau(j_0^2+j_1^2)\,\exp(-i(\Phi(m_f)+2\tau)), \qquad (6)\\
\Phi(m_f) & = & \frac{4\tau}{1-y^2}\,\frac{m_f^2}{(kq)}\Big[1+x_f^2\Big(1-j_0^2(\tau)\Big)\Big],\\
x_f^2 & = & -\,Q_f^2\,\frac{e^2a^2}{m_f^2},
\end{aligned}
$$

where $j_0(\tau)=\sin\tau/\tau$, $j_1(\tau)=-j_0'(\tau)$ are spherical Bessel functions, x_f^2 is the parameter of the wave intensity. It is easy to see, that the expressions for the amplitudes (5) and (6) have no divergences and are gauge-invariant, as they are expressed in terms of the electromagnetic field tensor of the photon $f_{\mu\nu}$ and the external field tensor $F_{\mu\nu}$.

Previously we have investigated the radiative decay $\nu_i \to \nu_j \gamma$ in a homogeneous magnetic field [10]. It is known that the weak field case with large dynamical parameter $\chi_f^2 = e^2 (p_1 F F p_1)/m_f^6$ corresponds to the crossed field limit. This circumstance may be a peculiar kind of test of the correctness of our calculations in the wave field, since the monochromatic wave also admits the crossed field limit ($\omega \to 0$ with fixed field strengths). As would be expected the amplitudes in these both cases really coincide. In fact, the amplitude $\mathcal{M}^{(+1)}$ in Eqn.(5), describing the ultrarelativistic neutrino decay $(E_\nu \gg m_\nu)$ in the crossed field limit are consistent with the corresponding expressions (8) of Ref. [10] ($\mathcal{M}^{(+2)} \to 0$ in this limit).

3 The decay probability of the ultrarelativistic neutrino

The decay probability $\nu_i \to \nu_j \gamma$ in the wave field

$$w = \sum_{n=-2}^{+2} w^{(n)} \tag{7}$$

in the ultrarelativistic limit ($E_\nu \gg m_\nu$) has the form:

$$\begin{aligned}
E_\nu w^{(-2)} &\sim O\Big(\alpha\, \frac{G_F^2 m_\nu^{10}}{m_e^4}\, x_e^4\Big), \\
E_\nu w^{(-1)} &\sim O\Big(\alpha\, \frac{G_F^2 m_\nu^{8}}{m_e^2}\, x_e^2\Big), \\
E_\nu w^{(0)} &\sim O\Big(\alpha\, G_F^2\, m_\nu^2\, m_e^4\, x_e^4\Big), \\
E_\nu w^{(+1)} &\simeq \frac{4\alpha}{\pi}\, \frac{G_F^2}{\pi^3}\, m_e^6\, x_e^6\, \Big|K_{ie} K_{je}^* - \frac{1}{2}\,\delta_{ij}\Big|^2 \\
&\times \int\limits_{-1}^{+1} dz \frac{1-z}{(1+z)^2}\, |J_1(m_e)|^2, \\
E_\nu w^{(+2)} &\simeq \frac{\alpha}{4\pi}\, \frac{G_F^2}{\pi^3}\, (p_1 k)\, m_e^4\, x_e^4\, \Big|K_{ie} K_{je}^* + \frac{1}{2}\,\delta_{ij}\, g_e\Big|^2
\end{aligned} \tag{8}$$

$$\times \int_{-1}^{+1} \frac{dz}{1+z} \left[\frac{(1-\xi)}{2} + \frac{(1-z)^2}{4} \; \frac{(1+\xi)}{2}\right] |J_2(m_e)|^2,$$

where $z = \cos\theta$, θ is the angle between the photon momentum $\vec{q}$ and the wavevector $\vec{k}$ in the center of the mass frame of the ν_j and γ. Consequently, in the ultrarelativistic limit $(qk) \simeq (p_1 k)(1+z)/2$ needs to be substituted in the expression (5) and (6). Notice that there is no singularity in the lower limit $z \to -1$ because the integrals J_1, J_2 tend to zero sufficiently fast. Only the contribution of the virtual electron in the loop is kept in the expressions (8). This is due to the fact that this contribution dominates over the others under consideration

$$E_\nu \omega < 10^{16} (eV)^2,$$

It should be pointed out that the decay probabilities (8) practically do not depend on the mass of the neutrino. Consequently, the radiative decay probabilities of a lighter neutrino into heavier one and of a heavier neutrino into lighter one are equal.

It is of interest to compare the expressions (8) with the well known decay probability $\nu_i \to \nu_j \gamma$ without the field [4]:

$$w_0 \simeq \frac{27\alpha}{32\pi} \, \frac{G_F^2 m_\nu^5}{192\pi^3} \, \frac{m_\nu}{E_\nu} \, \Big(\frac{m_\tau}{m_W}\Big)^4 |K_{i\tau} K^*_{j\tau}|^2. \tag{9}$$

This comparison shows that the very small GIM suppression factor $\sim (m_\ell/m_W)^4$ is absent in the probability of the radiative decay (8). Furthermore, there is no suppression caused by the smallness of the mass of the neutrino in the case of $n = 1, 2$. To illustrate the enhancing influence of the wave field on the decay probability $\nu_i \to \nu_j \gamma$ we present the numerical estimation of the ratio of the probability $\nu_i \to \nu_j \gamma$ from the high energy accelerator in the wave field of laser type to the decay probability in vacuum:

$$R = \frac{w}{w_0} \sim 10^{33} \Big(\frac{1eV}{m_\nu}\Big)^6 \Big(\frac{E_\nu \omega}{m_e^2}\Big)^5 \Big(10^3 x_e^2\Big)^2, \tag{10}$$

where the parameter of the wave intensity (1) can be represented in the following form:

$$x_e^2 \simeq 10^{-3}\Big(\frac{\mathcal{E}}{10^9 V/cm}\Big)^2\Big(\frac{1eV}{\omega}\Big)^2. \tag{11}$$

Such a significant enhancement of the decay probability $\nu_i \to \nu_j\gamma$ is, in our opinion, of great interest, even in a relatively weak electromagnetic field ($x_e^2 \sim 10^{-3}$). The results obtained in this work will be of use as to their application in astrophysics and cosmology. For example, the crossed process $\gamma \to \nu_i\tilde{\nu}_j$ of the photon splitting into the neutrino pair is possible in the wave field (the amplitude of this process is described by Eq. (5)). The probability of this process has the form:

$$\begin{aligned} w &= \frac{\alpha}{3\pi}\,\frac{G_F^2}{8\pi^3}\,\frac{m_e^4}{q_0}\,x_e^4\Big\{8m_e^2 x_e^2|J_1(m_e)|^2\,|K_{ie}K_{je}^* - \frac{1}{2}\,\delta_{ij}|^2 \\ &+ (qk)|J_2(m_e)|^2\,|K_{ie}K_{je}^* + \frac{1}{2}\,\delta_{ij}|^2\Big\}. \end{aligned} \tag{12}$$

It can be treated as an additional mechanism of the energy loss by stars etc.

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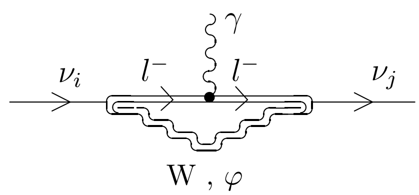
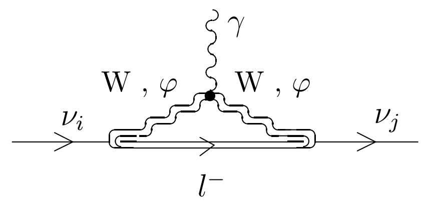


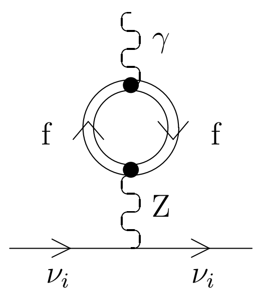
γ
ν_i
l^-
l^-
ν_j
W , φ

(a)



γ
W , φ
W , φ
ν_i
ν_j
l^-

(b)



γ
f
f
Z
ν_i
ν_i

(c)

Fig. 1.